Abstract

The frulllani integral has the expression

$$\int_0^\infty \frac{f(ax) - f(bx)}{x} dx \tag{1}$$

where $a, b > 0$ and $f : [0, \infty) \to \mathbf{R}$ be a continuously differentiable on $[0, \infty)$. It was named after the Italian Mathematician Giulano Frullani. There are some articles and references worked on the solution of (1). Some of them are generalized and some of them are not. But this paper focuses on the most popular result of the frullani Integral, i.e.,

$$\int_0^\infty \frac{f(ax) - f(bx)}{x} dx = \ln\left(\frac{a}{b}\right) [f(\infty) - f(0)]. \tag{2}$$

where $f(\infty) = \lim_{x\to+\infty} f(x)$ and $f(0) = \lim_{x\to 0^+} f(x)$. The purpose of this paper was to expose the classical proof of frullani Integral described in (2). Also, to gave an alternative proof of (2) different from the existing proof. And finally, to arrive at some corollaries as natural consequences of the alternative proof. The paper was mainly an expository. To understand the flow of the paper, theories and concepts in Advanced Calculus were needed. The alternative proof was started by stating and proving lemmas that were used on the flow of the proof. Also, another condition was applied to obtain the desired result. Consequences of the alternative proof were illustrated to evaluate some special integrals of frullani type.