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SOME BASIC PROPERTIES OF HADAMARD GROUPS

128000

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NOTATION

$\langle G, * \rangle$	a group G with the operation $*$
$M_n(F)$	algebra of $n \times n$ matrix
A_n	alternating group on n elements
D_n	dihedral group of order $2n$
G/N	factor group of G by N
$\text{Irr } G$	irreducible characters of G
$[G:H]$	index of H in G
$o(G)$	order of a group G
$o(a)$	order of an element a
$a b$	a divides b
$C(G/\sim)$	space of all class functions of G
$\text{stab}_G(l)$	stabilizer of l in G
S_n	symmetric group on n elements
$\text{tr}(\rho(g))$	trace of the matrix $\rho(g)$
\mathbb{C}	set of complex numbers
\mathbb{C}^*	set of nonzero complex numbers
\mathbb{Z}	set of integers
\mathbb{Z}_n	group of integers modulo n



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v

$\mathbb{Z}_n \setminus \{0\}$	group of integers modulo n excluding 0
D_a	set $\{d_1 a \mid d_1 \in D \text{ and } a \in G\}$
$D_1 D_2$	set $\{d_1 d_2 \mid d_1 \in D_1 \text{ and } d_2 \in D_2\}$
\mathbb{Q}	set of rational numbers
\mathbb{R}	set of real numbers
U_n	multiplicative group of n th roots of unity
e	identity element
I	identity matrix
\emptyset	null set
$\langle \phi, \lambda \rangle$	inner product
\cong	isomorphic to
\in	membership
ρ	representation of
Ω	set $\{1, 2, \dots, n\}$
\leq	subgroup inclusion
\triangleleft	normal subgroup
Σ	sum of
a^t	transpose of a



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ABSTRACT

Let G be a finite group of order $2n$ with a subset D and an element e^* such that $|D| = n$ and

$$(1) |D \cap Da| = \begin{cases} n & \text{if } a = e \\ 0 & \text{if } a = e^* \\ n/2 & \text{if } a \neq e, e^*, a \in G \end{cases}$$

$$(2) |Da \cap \{b, be^*\}| = 1 \text{ for any elements } a \text{ and } b \text{ of } G.$$

Then G is called an Hadamard group.

This thesis is a detailed study about some of the basic properties of Hadamard groups. This is based on the paper of Noboru Ito entitled "On Hadamard Groups" which appeared on Journal of Algebra, Volume 168, Number 3 on September 15, 1994.

