DE LA SALLE UNIVERSITY

ON SOME PROPERTIES OF CYCLIC TOURNAMENTS

A Thesis

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LIST OF NOTATIONS

$a \mid b$	a divides b, 22
a_{ij}	the (i,j)th entry of a matrix, 16
A(i,j)	the (i,j)th entry of a matrix, 68
A(n)	alternating group on n elements, 42
Aut(G)	set of all automorphisms of a group, 30
$\varphi(A)$	order of $N(v) \cap G(A)$, 96
C	permutation cycle (123...v), 62
$\mathbb{C}$	set of complex numbers, 19
$\langle C \rangle$	cyclic group generated by C, 78
C(H)	centralizer of H, 35
Cl(H)	conjugate class of H, 36
core(H)	core of H, 120
D_n	nth dihedral group, 20
e	identity element, 18
$\langle G, * \rangle$	group, 18
[G:H]	index of H in G, 27
G(A)	automorphism group of a tournament, 64

G/N	factor group of G by N, 28
$G(p)$	permutation group of order pu(p), 109
$id(x)$	indegree of x, 14
(i,i^*)	complementary pair, 70
$\phi:i\rightarrow j$	mapping of i into j, 76
I_v	identity matrix of order v, 85
K_ϕ	kernel f ϕ, 29
$\varphi(n)$	Euler totient function on n, 94
$N(H)$	normalizer of H, 35
$N(v)$	stabilizer of 1 in the normalizer $N(\langle C\rangle)$ of $\langle C\rangle$, 78
$o(G)$	order of G, 20
$O(1)$	outneighborhood of vertex 1, 68
$od(x)$	outdegree of x, 14
$orb_G(i)$	orbit of i under G, 43
P^{-1}	inverse of a permutation matrix P, 62
P^T	transpose of P, 62
$\mathbb{Q}$	set of rational numbers, 19
$\mathbb{R}$	set of real numbers, 19

$\mathbb{R}^+$	set of positive real numbers, 32
$\mathbb{R}\setminus\{0\}$	set of real numbers excluding 0, 32
$S(n)$	symmetric group on n elements, 41
$stab_G(i)$	stabilizer of i in G, 44
$U(n)$	group of all positive integers less than and relatively prime to n, 19
$u(p)$	odd portion of $p-1 = 2^m u(p)$, 109
$\|W\|$	cardinality of W, 114
$W(v)$	set of all subgroups of the symmetric group S(v) of odd orders containing C, 74
$\mathbb{Z}$	set of integers, 19
$\mathbb{Z}^+$	set of positive integers, 69
Z_n	group under integers modulo n, 19
$\in$	membership, 14
$\notin$	nonmembership, 16
$\subseteq$	set inclusion, 74
$\leq$	subgroup inclusion, 21
$\triangleleft$	normal subgroup of, 27
Ω	set { 1, 2, 3, ..., n }, 17
$\Longrightarrow$	implies that, 87

ABSTRACT

The thesis presents four main theorems on cyclic tournaments. The first deals with the problem of determining the size of any equivalence class of a tournament A in the set C(v) of all cyclic tournaments of order v. The form of an element of the set W(v) of all subgroups of S(v) of odd orders containing C = (123...v) as an automorphism group for some cyclic tournaments is introduced by the second proposition. This is extended to the form of a maximal element of W(v) which is demonstrated by the third theorem using the Polya composition operation. The last theorem discusses a way to determine if an element of W(v) is of the largest order by a certain linear order of odd primes.

The main results presented by Noboru Ito in the article "On Cyclic Tournaments" are amplified. Illustrations are provided to lend plausibility to the theorems. Related theorems and definitions needed in the subsequent arguments of the study but are not stated in Ito's paper are also presented.

Some of the primary properties of cyclic tournaments are proved in this study. In addition, a procedure to construct a cyclic tournament such that the automorphism group contains an element of W(v) is demonstrated.