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# DE LA SALLE UNIVERSITY

ON REGULAR AND 2-SCORED 2-ORTHOGONAL TOURNAMENTS

MA Thesis

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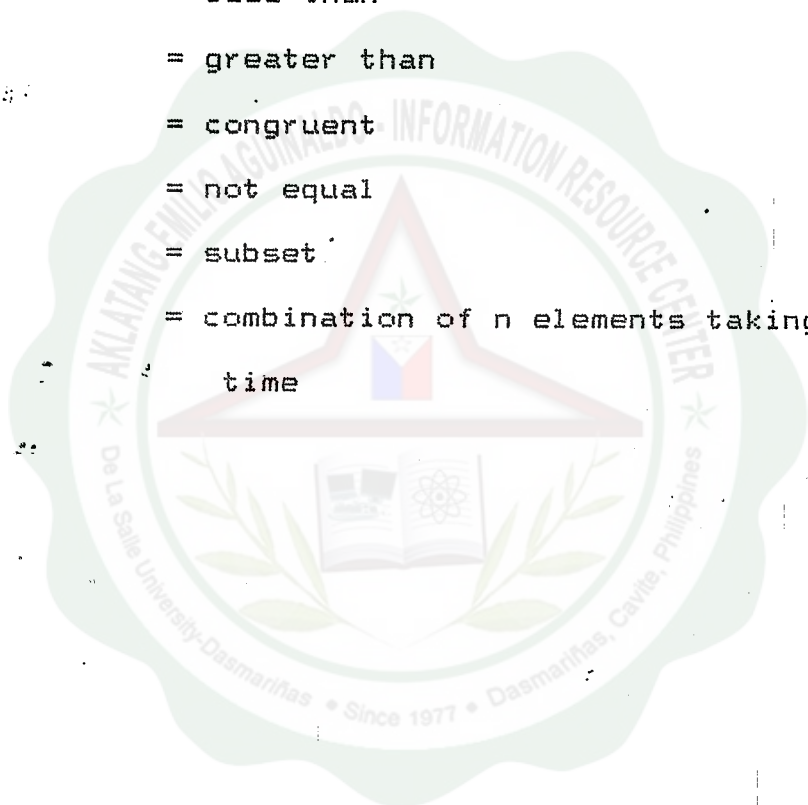


## LIST OF NOTATIONS

$\mathbb{R}$	= set of real numbers
$v_i$	= $i$ th vertex of $V$
$V$	= vertex set of a digraph $G$
$A$	= arc set of a digraph $G$
$[v_i, v_j]$	= undirected arc joining $v_i$ and $v_j$
$(v_i, v_j)$	= directed arc joining $v_i$ to $v_j$
$d(v_j)$	= degree of a vertex $v_j$
$id(v_j)$	= in-degree of a vertex $v_j$
$od(v_j)$	= out-degree of a vertex $v_j$
$a_{ij}   A(i, j)$	= an $i$ th row, $j$ th column element of matrix $A$
$A(G)$	= adjacency matrix of graph $G$
$A^t$	= transpose of matrix $A$
$I$	= identity matrix
$J$	= all-one matrix
$j$	= all-one row vector
$\Sigma$	= summation
$\forall$	= for all
$\in$	= element of
$\Rightarrow$	= implies
$A(i)$	= $i$ th row vector of $A$
$A^c$	= complement of $A$



$s(A(i))$	= score of $A(i)$
$(A(i), A(j))$	= intersection number of $A(i)$ and $A(j)$
$\mathbb{Z}$	= set of integers
$\mathbb{Z}^+$	= set of positive integers
$\leq$	= less than or equal
$<$	= less than
$>$	= greater than
$\equiv$	= congruent
$\neq$	= not equal
$\subseteq$	= subset
${}^C_n r$	= combination of $n$ elements taking $r$ at a time



## ABSTRACT

Let  $v$  be a positive integer such that  $v \equiv 3 \pmod{8}$ . Let  $A$  be a tournament of order  $v$ , then,  $A$  is 2-orthogonal if the product  $AA^t$  equals  $I$  where the multiplication is modulo 2,  $A^t$  is the transpose of  $A$  and  $I$  is the identity matrix.

This study presents two main theorems; the first shows the existence of regular 2-orthogonal tournaments of order  $v$  while the second shows the existence of 2-scored 2-orthogonal tournaments.

Aside from these two theorems, this paper also includes other theorems necessary in the study of 2-orthogonal tournaments relative to its score set. It presents all the proofs to the theorems and propositions as given in Noboru Ito's paper entitled "On 2-Orthogonal Tournaments" which will appear in the Proceedings 22nd Annual Meeting, Iranian Mathematical Society.

