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ON DISTANCE-REGULAR GRAPHS WITH $K_1 = K_j$

A Thesis

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by

IMELDA B. GARCIA

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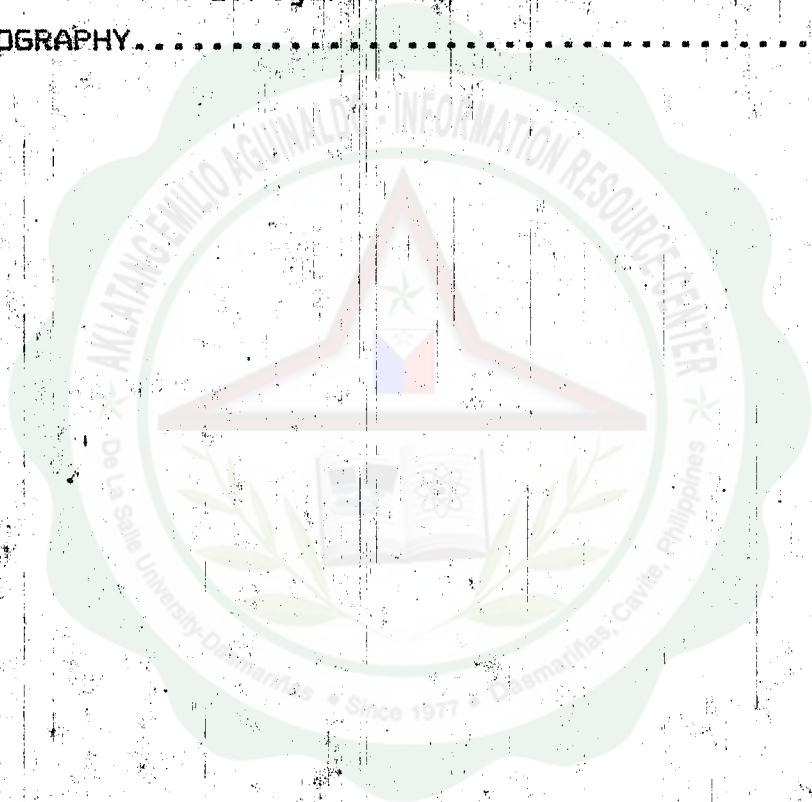
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LIST OF NOTATION

$\Gamma = (V(\Gamma), E(\Gamma)) =$ A GRAPH

$d(\Gamma) = d =$ diameter of Γ

$k = k(u) =$ valency of vertex u

$\Gamma_i(u) =$ set of points at distance i from a vertex u in Γ

$k_i = |\Gamma_i(u)|$

$D_j^i(u, v) = \Gamma_i(u) \cap \Gamma_j(v)$

$V(\Gamma) =$ vertex set of Γ

DRG = Distance Regular Graph

$E(\Gamma) =$ edge set of Γ

$n =$ order of Γ

$\partial(x, y) =$ distance between vertices x and y

$e(u) =$ eccentricity of u

$e(x, D_j^i) =$ number of edges from x to points in D_j^i

$p_{ij}^m =$ cardinality of $D_j^i(u, v)$ where u and v are at distance m from each other

$c_i = p_{i-1}^i$

$a_i = p_{ii}^i$

$b_i = p_{i+1}^i$

$\in =$ is an element of



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\notin = is not an element of

\subset = is contained in

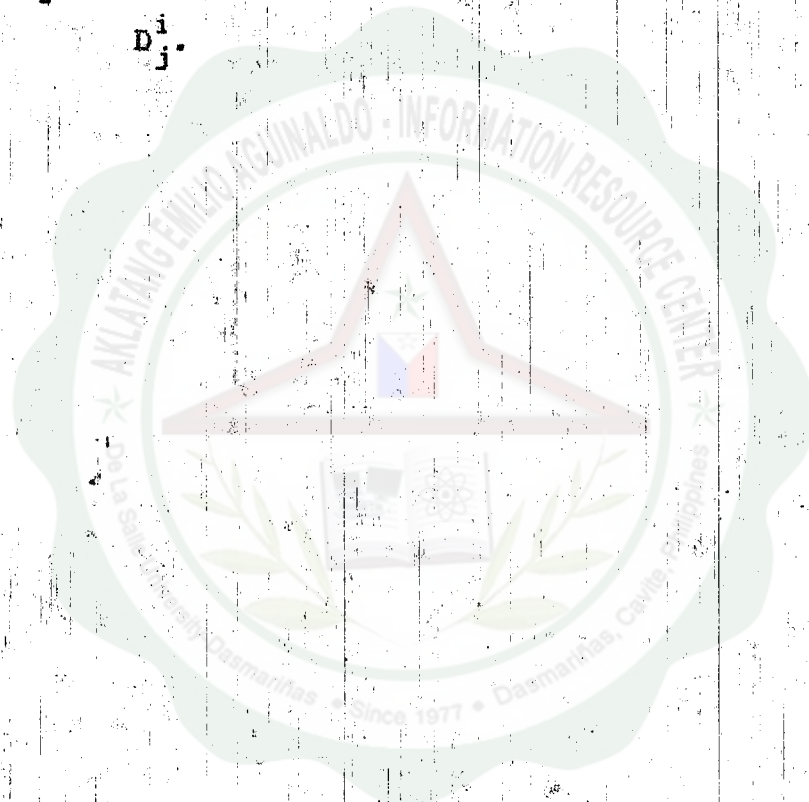
\cap = intersection

\cup = union

\Rightarrow = implies

\Leftrightarrow = if and only if

$e(x, D_j^i)$ = number of edges connected from x to points in D_j^i .



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ABSTRACT

Let Γ be a distance regular graph with diameter d and valency k . For nonnegative integers i and j with $0 \leq i+j \leq d$ and vertices u and v in Γ , let

k_i = number of points at distance i from u ,

k_j = number of points at distance j from v .

This thesis is a detailed study about distance regular graphs satisfying $k_i = k_j$. In particular, this paper aims to show the following:

a) The number of vertices at distance d from any vertex u in Γ is 1.

b) The number of points at distance e from u , with $i \leq e \leq j$, is constant.

c) If the number of points at distance j from u is not equal to the number of points at distance $(j+1)$ from u , then the points at distance d from u form a clique for any vertex u in Γ .

This thesis is based on the paper of Hiroshi Suzuki entitled "On Distance Regular Graphs with $k_i = k_j$ ".

