# DE LA SALLE UNIVERSITY

# TWO APPROACHES ON THE PROOF OF THE AMITSUR-LEVITZKI THEOREM

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A Thesis
Presented To
the Faculty of Graduate School
De La Salle University

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In Partial Fulfillment
of the Requirement for the Degree of
Master of Science in Mathematics

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by
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September 1991

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## ACKNOWLEDGMENT

The author wishes to express his gratitude to the following people who consistently helped him in the writing of his thesis:

Dr. Blessilda Raposa, his adviser, for her invaluable assistance and critical review of the entire paper,

Ms. Shirley Agatep, for her generosity to use the computer,

Officers and members of KG-SPCM, for their countless support and prayers,

Jose Clarito Covacha, for financial assistance and persistently urging the author to finish his thesis,

Nohree Q. Santos, for providing the author the needed inspirations and encouragements that have made the speedy writing of this thesis,

Florenda Bagorio, his mom and brothers, Broderick, Benjamin, Bertrand, for the unfailing cooperation and moral support,

And above all, to Almighty Lord, for without Him everything is impossible.

# CHAPTER V

## SUMMARY

We have presented here two methods of proving the Amitsur-Levitzki theorem. The graph theoretic proof was presented to give the readers the impression that there are several ways to prove a certain theorem. Inevitably, a reader would prefer a proof that is easy to understand, something that is less complicated. The simpler the proof is the better. The application of graph theory in proving Amitsur-Levitzki theorem has somehow minimized some complications. The employment of it has eliminated most of the complicated algebraic definitions merely by using much simpler graphic notions and by drawing a picture of the appropriate graph.

## CONCLUSION

In this paper, we were able to prove the following theorems:

1. If F is a field, then $M_m(F)$ satisfies the standard polynomial identity $s_{2m}$ .

2. Let G be a graph of order n, let A be the set of arcs of G such that $|A| = m$ and $m \geq 2n$. Let v and w be any fixed vertices of G. Then the number of unicursal paths $\omega$ from v to w with $\varepsilon(\omega) = +1$ is equal to the number of unicursal paths $\omega$ from x to y with $\varepsilon(\omega) = -1$.

Aside from this, we were able to give some basic properties of polynomial identities and consequently the standard polynomial identities. Furthermore, the necessary and sufficient conditions for the existence of the unicursal paths in a given graph were discussed in this paper.

## RECOMMENDATIONS

Our recommendations in this paper, would focus on the algebraic aspect, particularly on polynomial identities. The concept of polynomial identities seems to suggest some special considerations for further studies. Hence, we recommend the following topics for others to do some research on.

1. We obtained in this study the result if E is an algebra over F, contained in $M_m(F)$, then it satisfies a polynomial identity, in fact a standard identity $s_{2m}$. The question here concerns the converse, that is, can all polynomial identity algebras be imbedded in suitable matrix rings? If so, what are those polynomial identity algebras that are imbeddable in matrix rings over commutative rings?

2. Another topic that might be of interest is the concept of generalized polynomial identites. Let E be an algebra over F. A generalized polynomial over E is, roughly speaking, a polynomial in the noncommuting indeterminates $x_1, x_2, \ldots\ldots, x_n$ in which the elements of E are allowed to appear both as coefficients and as entries between the indeterminates. Hence, E satisfies a generalized polynomial identity, if there exists a nonzero generalized polynomial $f(x_1, x_2, \ldots\ldots, x_n)$ such that

$$f(\alpha_1, \alpha_2, \ldots\ldots\ldots, \alpha_n) = 0$$

for all $\alpha_i$'s $\in$ E. The problem here is to find the conditions that will ensure the existence of a generalized polynomial identity.